Linear Momentum and Newton’s Second Law

- **2nd Newton’s Law for any object:**
  \[ F = ma \]

- **Definition of acceleration:**
  \[ a = \frac{\Delta v}{\Delta t} \]

Hence, we can write Second Newton’s Law of Motion as:

\[ F = m \frac{\Delta v}{\Delta t} = m \frac{m \Delta v}{m \Delta t} = \Delta (mv) \]

- **Definition of linear momentum:**
  \[ p = mv \]

- **Change in linear momentum:**
  \[ \Delta p = m \Delta v = \Delta mv = \Delta (mv) \]

Second Newton’s Law of Motion in terms of linear momentum:

\[ F = \frac{\Delta (mv)}{\Delta t} = \frac{\Delta p}{\Delta t} \]

**Example 1:** Impulse

A car was moving with a speed of 60 km/h and crashed into a wall. The contact time is 0.2 s, a mass of the driver is 70 kg. Find the force acting on the driver during the crash.

1. **Conversion km/h to m/s:**

\[ 60 \text{ km/h} = 60 \left( \frac{1000 \text{ m}}{3600 \text{ s}} \right) = 16.67 \text{ m/s} \]

2. **A change in linear momentum (impulse) of the driver:**

\[ \Delta p = (\Delta v)m = (v_{\text{fin}} - v_{\text{init}})m = (0 - 16.67 \text{ m/s})(70 \text{ kg}) = -1166.9 \text{ kg} \cdot \text{m/s} \]

3. **A force acting on the driver:**

\[ F = \frac{\Delta p}{\Delta t} = \frac{-1166.9 \text{ kg} \cdot \text{m/s}}{0.2 \text{ s}} = -5834.5 \text{ N} \]

**Impulse**

Again: Second Newton’s Law of Motion in terms of linear momentum:

\[ F = ma = m \frac{\Delta v}{\Delta t} = m \frac{m \Delta v}{m \Delta t} = \Delta (mv) \]

To change a linear momentum of any object, a net force must act on it!

Let’s consider that a constant force acts on an object during time interval \( t \) :

\[ \Delta t = t_{\text{fin}} - t_{\text{init}} \]
\[ \Delta v = v_{\text{fin}} - v_{\text{init}} \]
\[ \Delta p = p_{\text{fin}} - p_{\text{init}} \]
\[ \Delta t = \frac{\Delta (mv)}{\Delta t} = F \]
\[ F \Delta t = \Delta (mv) \]

Impulse = change in linear momentum:

\[ F \Delta t = \Delta (mv) = \Delta p \]

**Conservation of Momentum**

- The forces that any particles of the system exert on each other particle of this system are called **internal forces**.
- The forces exerted on any part of the system by some object outside of the this system are called **external forces**.

If the total external force acting on the system is zero, the **total momentum of this system is conserved**.

The system consisting of two objects:

\[ p_{\text{total}} = p_A + p_B = \text{const} \]

\[ p_A = m_AV_A \]
\[ p_B = m_BV_B \]
Example 2: Conservation of Momentum (#1)

A rifle of mass \( m_R = 3 \text{ kg} \) fires a bullet of mass \( m_B = 5 \text{ g} \) horizontally with a velocity relative to the ground \( v_B = 300 \text{ m/s} \). What is the recoil velocity of the rifle, \( v_R \)? What is the final momentum of the bullet and rifle?

1. Conservation of the total momentum:

\[
p_{\text{total,init}} = m_B v_{B,\text{init}} + m_R v_{R,\text{init}} = 0
\]
\[
p_{\text{total,fin}} = p_{\text{total,init}} = 0
\]
\[
p_{\text{total,fin}} = m_B v_{B,\text{fin}} + m_R v_{R,\text{fin}} = 0
\]
\[
-m_B v_{B,\text{fin}} = m_R v_{R,\text{fin}} \quad \Rightarrow \quad v_{R,\text{fin}} = -\frac{m_B v_{B,\text{fin}}}{m_R}
\]

Example 2: Conservation of Momentum (#2)

2. The final momentum of the bullet:

\[
p_{B,\text{fin}} = m_B v_{B,\text{fin}} = (5 \times 10^{-3} \text{ kg})(300 \text{ m/s}) = 1.5 \text{ kg} \cdot \text{m/s}
\]

3. The final momentum of the rifle:

\[
p_{R,\text{fin}} = -p_{B,\text{fin}} = -1.5 \text{ kg} \cdot \text{m/s}
\]

4. The recoil velocity of the rifle:

\[
v_{R,\text{fin}} = -\frac{p_{R,\text{fin}}}{m_R} = -\frac{-1.5 \text{ kg} \cdot \text{m/s}}{3 \text{ kg}} = -0.5 \text{ m/s}
\]

ACT 1: Cart and Rain

Suppose rain falls vertically into an open cart rolling along a straight horizontal track with negligible friction. As a result of the accumulating water, the speed of the cart:

1) Increases
2) Doesn’t change
3) Decreases

The total momentum of the system (the cart and water) is conserved!

Because the mass of the system (the cart with water) increase, its speed will decrease. Note: water (rain) here doesn’t have horizontal velocity!

Spinning Figure Skater

Some figure skaters can make about 7 revolutions per second, or 420 revolutions per minute!

What is the difference between these two cases?

The radius of rotation of the skater’s arms!

Changing the radius of rotation of arms, the skater changes the angular momentum:

What is the angular momentum?